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HOW ABOUT VORTONS ?

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Abstract.

It has been suggested that cosmological dark matter may include a population of vortons (meaning small centrifugally stabilised cosmic string loops) as an outcome of (non-standard) electroweak symmetry breaking. The implications for this conjecture of recent theoretical progress on superconducting string theory are discussed, particularly in relation to problems of stability. It is tentatively concluded that if the underlying field theory provides a carrier field with mass not too far below that of the relevant Higgs mass (which may be a few hundred G.e.V.) electroweak string formation may conceivably produce enough vortons (with an extended mass distribution peaking perhaps at several hundred T.e.V.) to provide a marginally significant dark matter contribution.

1. How are they formed?

Many of the most commonly considered theories of G.U.T. and (non standard) electroweak symmetry breaking predict the formation of vacuum vortex defects[1]. If it is of the “local” or “gauged” type, its energy will be concentrated within a microscopic radius $r_x \approx m_x^{-1}$ where (using units such that $\hbar = c = 1$) m_x is the Higgs mass scale associated with the symmetry breaking, so that on a macroscopic scale the vortex can be represented by a thin “cosmic” string model. (The present discussion is not concerned with “global” vortices, with logarithmically divergent energy, for which a thin string description is less accurate but still useful.) When a finite loop of such a string is produced (whether during the symmetry breaking phase transition, or by a transconnection process later on) one expects it to start in a randomly excited state, whose energy will then dissipate by background drag and radiation. It will thus be damped down either until it reaches a “vorton” state, meaning a stationary or quasi stationary configuration[2][3] in which the energy is minimised with respect to any parameters that are effectively conserved during this process, or else until entirely disappears.

The present contribution updates an early discussion of this topic at previous “Moriond astrophysics” meeting, in which the term “cosmic ring” was used as an alternative to “vorton” [4][5][6][7] to express the property of circularity that was naively taken for granted until it was discovered[8][9] that non circular equilibrium states also exist, at least in the strict string limit $r_x \ll \ell$ where ℓ is the loop circumference.

The possibility of vorton formation originally first pointed out by Davis and Sheldahl[2][3], who were the first to fully appreciate the implications of Witten’s revelation[10] of the likely existence of mechanisms that can easily allow the occurrence of currents on cosmic strings. Earlier work on cosmic strings assumed that the underlying vortices were of the simplest Nielsen-Olesen type which are describable on a macroscopic scale by the longitudinally Lorentz invariant Goto-Nambu string model[1]. The absence of internal structure in this degenerate model makes the centrifugalsupport mechanism inoperative, and so excludes the existence of any vorton states.

Before the importance of the centrifugal support mechanism was recognised at last[2][3], several workers investigated the possibility of what were termed “cosmic springs”, meaning magnetostatically supported ring configurations[11][12][13] that unlike vortons were postulated to be non - revolving (strictly static as opposed to stationary). Their existence depended on the supposition that the Witten current is electromagnetically coupled, which is very plausible, but it was soon realised that they could hardly be even ephemerally viable under natural conditions if the relevant coupling constant has its usual very small value $e^2 \simeq 1/137 \ll 1$, because taken by itself the magnetic force would be far too weak to balance string tension except in very weakly curved configurations whose radius would have to be far too large for it to be realistic to consider them as isolated systems. It has however been shown by Peter[14] that although the magnetostatic effect can be expected to cause at most a very small modification of the radius of a circular vorton state (just slightly breaking the degeneracy that allows the existence of arbitrarily “bent” equilibrium states[8][9] for neutral string loops), on the other hand electrostatic support (a possibility that was overlooked in the earlier work) may be more important.

2. How rapidly do they revolve?

One reason why it was not immediately realised that the dominant effect of Witten type currents in vacuum vortices would be mechanical rather electromagnetic was the unavailability at that time of a suitable string model for describing their macroscopic behavior. Some of the first attempts to evaluate electromagnetic effects simply continued to represent the vortices as simple Goto Nambu strings in which the currents were postulated to evolve as if the effect of their inertia on the motion of the string were entirely non-existent[15]. Most of the earliest attempts to take account of such mechanical effects relied[16][17] on a model constructed as a particular kind of linearised perturbation of the Goto-Nambu model that had been proposed at the outset for use in the weak current limit by Witten himself[10]. A minority of earlier workers took up the rather more canny suggestion[18] that a conducting string model previously introduced in a very different physical context by Nielsen[19] should be used.

The consideration that none of these special models could deal with the predicted effect[10] of current saturation (sometimes referred to as “quenching”) motivated the development[20][21] of a more general category of elastic string models in which current dependence of an appropriately non-linear nature could be allowed for. This category consists of models governed by 2-dimensional (string worldsheet) action integral specified by a Lagrangian scalar \mathcal{L} that is given as some (generically non-linear) function of a surface momentum covector \tilde{p}_μ that is constructed as the tangentially projected gauge covariant derivative of a scalar field ϕ . In the application to Witten’s[10] bosonic superconducting model, ϕ is to be interpreted as the phase of the carrier condensate in the vortex. In any model of this category there will be a corresponding dynamically conserved surface current, $c^\mu = \kappa \tilde{p}^\mu$ with $\kappa = -2d\mathcal{L}/d\tilde{w} > 0$ where $\tilde{w} = \tilde{p}_\mu \tilde{p}^\mu$, and a corresponding surface stress momentum energy density tensor, which will be expressible[7][8][9] in terms of the fundamental tangential projection tensor η^μ_ν of the worldsheet simply as $T^\mu_\nu = c^\mu \tilde{p}_\nu + \mathcal{L} \eta^\mu_\nu$.

The non zero eigenvalues of T^μ_ν are the Lagrangian \mathcal{L} and its dynamical conjugate $\Lambda = \mathcal{L} - \kappa \tilde{w}$ whose negatives are identifiable with the rest frame energy density U and the string tension T according to the specification $U = -\mathcal{L}$, $T = -\Lambda$, in the “magnetic case”, $\tilde{w} > 0$, for which the current is spacelike, and $U = -\Lambda$, $T = -\mathcal{L}$ in the “electric” case, $\tilde{w} < 0$, for which the current is timelike. They will be functionally related in such a way that their difference is given by $U - T = \tilde{\kappa} |\tilde{w}|$. The ensuing characteristic speeds c_E and c_L of extrinsic “wobble” perturbations and sound type longitudinal “wobble” perturbations respectively will be given [21] by $c_E^2 = T/U$, $c_L^2 = -dT/dU$.

The availability of this category of elastic string models makes it possible[6][7] to derive many important properties of vortons even in the absence of detailed knowledge of the appropriate equation of state. As a special case of a more general study allowing for a general stationary gravitational or electromagnetic background field[8] it was established that in the absence of any background fields an equilibrium configuration for any such string model must (as one might have guessed) be *straight* (which is evidently impossible for a closed string loop) except in the special *transcharacteristic* case characterised by $v^2 = c_E^2$, where v is the longitudinal running velocity as determined by the timelike surface energy eigenvector that is aligned with c^μ in the “electric” regime where the current is timelike, and that is orthogonal to c^μ in the “magnetic” regime where the current is spacelike (which was the only possibility

considered in the earliest work). This centrifugal force balance condition evidently implies that a vorton must have a revolution period given by $\ell\sqrt{U/T}$ where ℓ is its circumference. When the current is electromagnetically coupled, the transcharacteristic running condition is modified to become a subcharacteristic running condition that (assuming circularity, which is presumably justified in this case) has been found by Peter[14] to have a form expressible in terms of the electric current intensity $I = e\sqrt{c^\mu c_\mu}$, using a suitably adjusted definition of the effective radius r_σ of the current distribution, as $v^2 = (T - I^2 \ln\{\ell/r_\sigma\}) / (U + I^2 \ln\{\ell/r_\sigma\})$.

The specification of the running velocity v determines the ratio of the integral quantum numbers characterising the vorton, which can be defined to be the phase winding number N and the total particle number Z associated with the current (so that when it is electromagnetically coupled the total electric charge will be $Q = eZ$), according[6][7] to the specification $Z/N = 2\pi\kappa v$ in the “magnetic” case $\tilde{w} > 0$ and $Z/N = 2\pi\kappa/v$ in the “electric” case $\tilde{w} < 0$. The equilibrium condition will thus be expressible in a form that is valid for either case by

$$\frac{|Z|}{|N|} = 2\pi\kappa \sqrt{\frac{\Lambda + \tilde{w}\kappa^2 e^2 \ln\{\ell/r_\sigma\}}{\mathcal{L} - \tilde{w}\kappa^2 e^2 \ln\{\ell/r_\sigma\}}}.$$

The modification arising from a non zero electromagnetic coupling constant $e \simeq 1/\sqrt{137}$ in the foregoing formula can be expected to be very small in the “magnetic” case, i.e. when the current is spacelike due to the saturation limit $c^\mu c_\mu \lesssim m_\sigma^2$ due to the “quenching” effect that was predicted by Witten[10] and confirmed by more detailed investigations such as that of Babul, Piran, and Spergel[22], independently of whether the current is electromagnetically coupled or not, where $m_\sigma \approx 1/r_\sigma$ is the mass of the carrier field whose condensation on the string supports the current, since it is usually assumed that this mass is at most of the order of the Higgs mass, $m_\sigma^2 \lesssim m_x^2$, which itself determines the magnitude of the energy density and tension, $U \approx T \approx m_x^2$ in this case. If the carrier mass is relatively small, the corresponding running speed will have to be relativistic, with Lorentz factor $\gamma = 1/\sqrt{1 - v^2}$ given by $\gamma \gtrsim m_x/m_\sigma$.

To describe the very different situation that arises in the less familiar “electric” case, it is helpful to consider the specific form of the equation of state, as derived in the first numerical investigation of the timelike current regime on a Witten vortex, by Peter. The computed form of the Lagrangian in the absence of electromagnetic coupling[23][24] has a form that has recently been found to be expressible rather accurately by an analytic formula given[25] in terms of a pair of constant mass parameters m and m_\star , and of a dimensionless normalisation constant κ_0 (which is typically of order unity), by $\mathcal{L} = -m^2 + (1 - \tilde{\chi}/m_\star^2)^{-1} \tilde{\chi}/2$ with $\tilde{\chi} = -\kappa_0 \tilde{w}$, which gives $\kappa = \kappa_0 (1 - \tilde{\chi}/m_\star^2)^{-2}$, in which $m \approx m_x$ and $m_\star \approx m_\sigma$, so that one expects to have $m_\star^2 < 16m^2$. Subject to this condition, the allowed range of the variable $\tilde{\chi}$ will have a lower limit $-m_\star^2/3 < \tilde{\chi}$ where $c_L^2 \rightarrow 0$ in the “magnetic” regime, and an upper limit $\tilde{\chi} < 2m^2 m_\star^2 / (2m^2 + m_\star^2)$ where $c_E^2 \rightarrow 0$ in the “electric” regime. The limit (attributable to “quenching”) in the magnetic regime imposes a corresponding limit $|Z/N| \gtrsim 2\pi\kappa_0$ on the values of the quantum numbers Z and N that are compatible with a vorton equilibrium state. When the ratio m/m_\star is small as one expects, the “wiggle” speed c_E will be not only relativistic but also supersonic over most of the allowed range for χ but it will become subsonic, $c_E < c_s$, when $\tilde{\chi} > (4m^2 - m_\star^2)/(4m^2 + 2m_\star^2)$, which is attained for large values of the charge to winding number ratio, $|Z/N| \gtrsim 8\pi\kappa_0 (m_x/m_\sigma)^5$.

When the current is electromagnetically coupled, the equation of state is modified in a manner that still needs further investigation, but that has been shown by Peter[26] to be negligible as far as the “magnetic” case is concerned, and that seems likely remain unimportant even in the electric case so long as the charge density remains well within the limit $\sqrt{|c^\mu c_\mu|} \lesssim m_\sigma/e^2$ set by the threshold beyond which charge leakage is to be expected. However if $m_\sigma^2/m_x^2 \lesssim 2e \simeq 1/6$ such modifications might be expected to become important when the charge density approaches the upper limit $|c^\mu c_\mu| < 2m^2(1 + 2m^2/m_\star^2)^3$ that would be allowed in the absence of coupling.

3. How small can they be?

For the string description to be valid, the loop circumference ℓ must substantially exceed the effective radius $r_\sigma \approx 1/m_\sigma$ of the current carrying sheath that surrounds the defect core with smaller radius $r_x \approx 1/m_x$. In terms of this value $\ell \gg r_\sigma \gtrsim r_x$, the mass M and the maximum value for the angular momentum J of a stationary string loop will be given[6][7] by $M = \ell(U + T)$ and $J \leq |ZN| = \ell^2 \sqrt{UT}/2\pi$.

Previous quantitative estimates[2][3][4][5] were restricted to the relativistically revolving case, which is the only possibility for vortons of “magnetic” type, i.e. those in which (whether or not there is electromagnetic coupling) the current is spacelike, and for which the Lorentz factor is bounded below by $\gamma \gtrsim m_x/m_\sigma$. For such relativistic (and therefore, by the preceding analysis, supersonic) vortons, one obtains $\sqrt{|ZN|} \simeq \ell m/\sqrt{2\pi} \approx \ell m_x$, and $M \simeq 2\ell m^2 \approx 2m_x \sqrt{|ZN|}$, in which the integers Z and N must be of comparable magnitude, $|Z/N| \simeq 2\pi\kappa \approx 2\pi\kappa_0$, and not too small in order to satisfy the requirement that ℓ/r_σ should be reasonably large.

In the “electric” case, for which (whether or not there is electromagnetic coupling) the current is timelike, there is also the possibility of a qualitatively different and so far little studied kind of vorton state whose revolution speed v is small compared with that of light, and can even fall below the soundspeed c_L . This arises because although the possibility of the tension tending to zero (a so called “spring” limit) has been excluded for reasonable parameter values in so far as spacelike currents are concerned[24], it does occur (at least in the absence of electromagnetic coupling) for timelike currents. In this low tension limit, $T \rightarrow 0$, the formula[25] quoted above gives $U \rightarrow 2m^2(2m^2 + m_\star^2)/m_\star^2$, from which one obtains the asymptotic relations

$$\frac{|Z|}{|N|} \sim 2\pi\kappa_0 (2m^2/m_\star^2)^{5/2} \sqrt{\frac{2m^2 + m_\star^2}{T}}, \quad |ZN| \sim \frac{\ell^2 m}{2\pi m_\star} \sqrt{2(2m^2 + m_\star^2)T}.$$

In this limit, $|Z/N| \gg 1$ one thus obtains $\ell \sim |Z|(m_\star/m)^3/\sqrt{\kappa_0(2m^2 + m_\star^2)}$. This will be consistent with the requirement $\ell \gg r_\sigma$ provided $|Z| \gg (m_x/m_\sigma)^4$, which (since $|N|$ cannot be less than one) will automatically be the case if the subsonicity condition $|Z/N| \gtrsim (m_x/m_\sigma)^5$ is satisfied. The relatively high quantum numbers needed for such slowly revolving vortons will be statistically harder to obtain than those needed for the more familiar relativistic kind, but for reasons discussed in the next section, the subsonically rotating ones may be the kind that is most durable.

4. How long can they last?

Having recognised that there is a natural process whereby vortons can be formed as an end product of energy dissipation by a dynamically evolving string loop, the next question one needs consider is that of their durability, a subject that needs a lot more work before firm conclusions can be drawn. In a systematic approach to this problem, the first thing to be checked is whether the equilibrium states are dynamically stable within the framework of the simple uncoupled string description described above. A general analysis of this problem has recently established[27] that an isolated circular string loop equilibrium state will always be dynamically stable if its running speed $v = c_E$ is subsonic or transonic. According to the preceding analysis, subsonic vortons need $|Z/N| \gtrsim (m_x/m_\sigma)^5$. For lower values of the charge quantum number $|Z|$, the vorton will fall in the supersonic range ($c_E > c_L$) in which it has been found[27] that some unstable states occur. A closer examination[28] has however shown that while these can only have an ephemeral existence (with lifetimes rarely more than a few times the light crossing time for the loop), there are plenty of other supersonically rotating vorton states that will be dynamically stable.

After this confirmation that many dynamically stable vorton string states are available, one of the next questions to be considered is the possibility of “secular” instability on much longer timescales, particularly due to electromagnetic coupling which, despite its relative weakness might have a significant cumulative effect in the long run. A well known example of such “secular” instability is provided by rotating perfect fluid neutron star models, which it is caused by gravitational radiation reaction in modes whose relative propagation speed is slower than that of the stellar rotation[29]. Although no analogous (relatively counterrotating but nevertheless forward rotating) modes exist in subsonically rotating states, they do exist in the more easily obtainable supersonic vorton states. In vortons whose current is not electromagnetically coupled and for which only relevant radiation mechanism is gravitational, one might expect that (as is the case in practice for neutron stars) the growth of such an instability would be too slow for it to be astrophysically relevant – at least for lightweight (electroweak unification) strings if not for heavyweight (G.U.T.) strings. However such instability might rapidly eliminate all electromagnetically coupled vortons except or the subpopulation in the subsonic range, so that after a cosmologically the only survivors of the less massive supersonic type would be neutral. (A contrary effect of electromagnetic coupling that might however be relevant[11] is its tendency to enhance stability by reducing the running velocity v slightly below the wiggle velocity c_E , which will tend to broaden the range of subsonic states.

Another kind of instability mechanism that was discussed when the concept of a vorton was first introduced[2][3][30], but that has not yet been the subject of a careful mathematical investigation, is a quantum barrier tunnelling process involving spontaneous emission of the carrier particles (with mass m_σ). Experience with nuclear physics shows that timescales for such processes are extremely difficult to estimate because they are exponentially dependent on quantities that are very sensitive to details of the internal structure, so that the question of whether the lifetime is microscopically short or cosmologically long can easily be affected by quite small uncertainties in the parameters of the underlying model. The crudest naive estimate for the decay timescale τ for a typical tunnelling process is to suppose that, in units of the relevant dynamical timescale, presumably of order of the Higgs timescale m_x^{-1} , it will be given by the exponential of a number that is roughly the number of wavelengths involved

in crossing the barrier. For a spin down process involving cancellation of linear momentum losses from opposite sides of the vorton, it is reasonable to guess that the relevant barrier width is of the order of the vorton diameter ℓ/π while the relevant wavelength will be that determined by the carrier field m_σ . According to this simple line of reasoning, the decay lifetime for a vorton of the more easily obtainable relativistic type will be given in very rough order of magnitude by $\ln\{m_x\tau\} \approx m_\sigma\ell/\pi \approx \sqrt{|ZN|}m_\sigma/m_x$.

According to more intricate but (because it involves questionable assumptions at intermediate stages) not obviously more convincing order of magnitude estimation by Davis[30], the factor m_σ/m_x on the right of the naive formula of the preceeding paragraph should be replaced by $(m_\sigma/m_x)^3$. If this were correct then for small values of m_σ/m_x a vorton would decay much more rapidly than predicted by preceeding estimate. However pending more sophisticated calculations (whose results are likely to be highly dependent on detailed assumptions about the parameters of the underlying field theory) to determine what is really most appropriate, the Ockham principle suggests that the simpler formula should perhaps be preferred. In any case if m_σ is not much smaller than m_x the uncertainty about the numerical coefficient will be just as important as the question of the extra factor $(m_\sigma/m_x)^2$. If m_σ is nearly the same as m_x , the two naive estimates agree in predicting that in order for the vorton to survive to the present day one needs something like $\sqrt{|ZN|} \gtrsim 10^2$. However for $m_\sigma/m_x \simeq 10^{-1}$ my naive provisional estimate of the requirement for survival to the present day is just changed to $\sqrt{|ZN|} \gtrsim 10^3$, whereas the much cited Davis estimate[3][30][31] would require $\sqrt{|ZN|} \gtrsim 10^5$.

The feasibility of the quantum emission processes to which such estimates apply is presumably dependent on the availability of suitable perturbed destination states of reduced energy for the vorton. However the classical analysis (and experience with ordinary laboratory superfluidity) suggests that, at least in the bosonic case, the existence of such reduced energy destination states depends on the flow speed of the condensate being supersonic. This makes it reasonable to conjecture that the subsonically rotating vorton states in the “electric regime”, would in fact be entirely stable for all practical cosmological purposes. (They would in principle still be able to decay a coherent manner, but this would require a simultaneous transition of nearly all the quanta involved, so that the marginally sufficient proportionality factor $\sqrt{|ZN|}$ in the estimate for $\ln\{m_x\tau\}$ would presumably need to be replaced a much larger safety factor, perhaps of the order of $|ZN|$).

5. How many are to be expected?

The essential preliminary to formation of a vorton is the creation of its dynamical string loop precursor, either at the time of cosmic string formation or by a subsequent string transconnection process. According to the standard picture[31][32][33] of cosmic string formation by the Kibble mechanism when the cosmological temperature Θ drops below the value $\Theta_x \approx m_x$ determined by the relevant Higgs mass scale, the ensuing dynamics will initially be controlled by the damping effect of the ambient background, which will rapidly smooth out short wavelength structure. However structure characterised by a curvature radius R large compared with the relevant damping scale τ say will be overdamped, and hence will be able to survive over a Hubble timescale $t = H^{-1}$ where $H \approx \sqrt{G} \Theta^2$ if and only if $R \gtrsim \xi$ where $\xi \approx \sqrt{\tau t}$. The time scale of the damping (due to drag by the thermal background) will be given by $\tau^{-1} \approx \beta \Theta^3 m_x^{-2}$ with a dimensionless drag coefficient β that depends on the underlying field theory[34] but that is expected to be of order of unity, which implies that the smoothing length scale will be given by $\xi \approx G^{-1/4} m_x \Theta^{5/2}$. So long as the temperature remains above a critical value $\Theta_* \approx \sqrt{G} m_x^2$ below which τ exceeds the Hubble time (so that drag becomes negligible and radiation reaction takes over as the dominant damping mechanism) any lengthscale $R \gtrsim \xi$ will be associated[33] with a corresponding number density $n \approx R^{-3}$ of wiggly (but, due to the overdamping, not wriggly) string loops of average length $L \approx R^2/\xi$.

It is during this overdamping epoch characterised by $\sqrt{G} m_x^2 \lesssim \Theta \lesssim m_x$ that the condensation of the carrier field at a critical temperature $\Theta_\sigma \approx m_\sigma$ is presumed to occur. The crucial step[2][3] in the formation of proto-vortons is a random walk type process whereby one expects that Z and N will acquire a Gaussian distribution with mean square value given by the ratio of the loop circumference to the thermal fluctuation scale, i.e. $\langle Z^2 \rangle \approx \langle N^2 \rangle \approx m_\sigma L$. The minimum initial loop circumference needed for producing vortons characterised by conserved numbers at least the magnitude of given values $|Z| \gtrsim |N|$ with reasonable efficiency will thus be given roughly by $L \gtrsim Z^2/m_\sigma$. Since the smoothing length at this time will be given by $\xi_x \approx G^{-1/4} m_x m_\sigma^{-5/2}$, this means that to obtain $Z^2 \gtrsim (G m_x^2)^{-1/4} (m_x/m_\sigma)^{3/2}$ one needs a corresponding minimum curvature length scale $R \gtrsim G^{-1/8} m_x^{1/2} m_\sigma^{-7/4} |Z|$.

If a fraction f of such loops survives in the form of vortons at a later epoch, their number density will be given in terms of this initial curvature length scale R by $n \approx f R^{-3} (\Theta/\Theta_\sigma)^3$ due to the cosmological expansion as the universe cools from the carrier condensation temperature Θ_σ to the lower temperature Θ at the later epoch under consideration. The corresponding cosmological mass density contribution will be $\rho \approx M n$ where M represents the typical individual mass energy value for such vortons, which will be given by $M \approx m_\sigma |Z|$ for the supposedly more durable slowly rotating kind needing a large value of $|Z|$ and by $M \approx m_\sigma |Z|$ for the initially more numerous relativistic kind, so that for the latter one obtains $\rho \approx f G^{3/8} m_x^{-1/2} m_\sigma^{9/4} Z^{-2} \Theta^3$.

It is of particular interest to evaluate the corresponding cosmological closure fraction $\Omega = \rho/\rho_c$ as evaluated with respect to the minimum mass density compatible with cosmological closure (according to the standard Einstein theory in the absence of a cosmological constant) which is given by $\rho_c \approx m_c \Theta^3$ where m_c is the required mass energy per black body photon which is of the order of 10^2 e.V. , i.e. $\sqrt{G m_c^2} \approx 10^{-26}$. The preceding formulae

provide the estimate

$$\Omega \approx f(Gm_x^2)^{3/8} \left(\frac{m_\sigma}{m_x} \right)^{9/4} Z^{-2} \frac{m_x}{m_c}$$

for the contribution from vortons with $|Z| \gtrsim |N| \gtrsim (Gm_x^2)^{-1/8} (m_x/m_\sigma)^{3/4}$ if they are of the relativistic kind, while if only the subsonic kind survives the final factor m_x/m should be replaced by the rather smaller factor m_σ/m_c , a change that is not very important in view of the other uncertainties (such as the values of the numerical “order of unity” numerical factors κ_0 and β). It can be seen that as $|Z|$ is increased above this smoothing limit the vorton spectrum decreases rather steeply. On the other hand the spectrum levels off for values below this limit so that the total subsonic contribution for moderate and small quantum number values will be given just by $\Omega \approx f(Gm_x^2)^{5/8} \left(m_\sigma/m_x \right)^{11/4} m_x/m_c$. The typical mass of a vorton in this sub population near the lower cut off limit (which would constitute the bulk of the population if f remained large for such low quantum number values) will be given by $M \approx m_x(Gm_x^2)^{1/8} (m_\sigma/m_x)^{1/4}$.

The enormous value $m_x/m_c \approx 10^{23}$ obtained for the final factor of the preceeding formula in the G.U.T. case makes it hard to see how the heavyweight string scenario can avoid excessive vorton production if there is any carrier field with mass m_σ that is even remotely comparable with the Higgs mass. Even for lightweight strings, with $m_x/m_\sigma \approx 10^{10}$, the value obtained for Ω would not be entirely negligible compared with unity if m_σ were nearly as large as m_x and if the survival fraction f is reasonably large. In order for f to avoid being negligibly small Z must be large enough to allow long term survival, which would seem likely to require at least $Z \gtrsim 10^2$ for avoidance of decay by quantum tunnelling in the rapidly revolving case, and $Z \gtrsim (m_x/m_\sigma)^5$ for stabilisation in a subsonically rotating state, which is a more stringent requirement if m_σ/m_x is very small. However provided $m_\sigma/m_x \gtrsim 10^{-1}$ even this requirement will be satisfied by most of the loops in the distribution, which (in the electroweak case characterised by $\sqrt{Gm_x^2} \approx 10^{-16}$) will peak at a smoothing limit given $Z \approx 10^4$.

Most such vortons would be on the high side of the multi T.e.V. mass range that has been envisaged for hypothetical ultramassive dark matter particles (known, if they are charged, as “CHUMPs” or “CHAMPS”). The expectation that their density would be too low for them to contribute much towards cosmological closure means that the formation of electroweak vortons cannot yet be ruled out by direct detection limits[35][36][37] or even by the indirect considerations[38] that place even more severe constraints on their abundance unless they are of the electrically uncoupled kind. The best chance of detecting a sparse but electrically coupled vorton distribution might be as cosmic rays. In particular, it has been suggested that rare events of extremely high energy that are hard to account for in terms of the usual acceleration mechanisms [39] might plausibly be attributed to vortons[40].

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